$$= \frac{-\frac{x}{\sin t \cos t} \cdot \frac{2x}{a^2} \tan^2 t + \frac{y}{\sin t \cos t} \cdot \frac{2y}{b^2} \cot^2 t}{\sqrt{\left(\frac{2x}{a^2} \tan^2 t\right)^2 + \left(\frac{2y}{b^2} \cot^2 t\right)^2}}$$

$$= \frac{a^2 y^2 \cot t \operatorname{cosec}^2 t + b^2 x^2 \tan t \sec^2 t}{x^2 b^4 \tan^4 t + y^2 a^4 \cot^4 t}$$ Ans.

Ex-13 / 29 Determine the acceleration of a fluid Particle of fixed identity for the velocity field.

$$q = i A x^2 y + j B y^2 z t + k C z t^2$$

Solⁿ: since $q = iu + jv + kw = iAx^2 y + jBy^2 zt + kCzt^2$

$$u = Ax^2 y, \; v = By^2 zt, \; w = Czt^2$$

The acceleration f of the fluid Particle is Given as

$$f = \frac{\partial q}{\partial t} + u \frac{\partial q}{\partial x} + v \frac{\partial q}{\partial y} + w \frac{\partial q}{\partial z}$$

$$f = jBy^2 z + k2Czt + Ax^2 y (i2Axy) + By^2 zt (iAx^2 + j2Byzt) + Czt^2 (jBy^2 t + kCt^2)$$

$$\Rightarrow f = A(2Ax^3 y^2 + Bx^2 y^2 zt) i + B(y^2 z + 2By^3 z^2 t^2) j + C(2zt + Czt^4)$$

Ans.

which determine the acceleration of a fluid Particle.

— o —

Let r be the radius of the cavity and v be the velocity at any time t. then

$$r' = r \;,\; v' = v \;,\; P = 0$$

$$-\frac{f'(t)}{r} + \frac{1}{2}v^2 = \frac{2\mu}{r^{1/2}}$$

Again $\quad f(t) = r^2 v$

diff. we have

$$f'(t) = 2r\frac{\partial r}{\partial t}v + r^2\frac{\partial v}{\partial t}$$

$$= 2rv^2 + r^2\frac{\partial v}{\partial r}\cdot\frac{\partial r}{\partial t}$$

$$= 2rv^2 + r^2 v\frac{\partial v}{\partial r}$$

or $\quad -\left(2v^2 + rv\frac{dv}{dr}\right) + \frac{1}{2}v^2 = \frac{2\mu}{r^{1/2}}$

or $\quad 2v^2 + rv\frac{dv}{dr} + \frac{1}{2}v^2 = -\frac{2\mu}{r^{1/2}}$

or $\quad rv\frac{dv}{dr} + \frac{3}{2}v^2 = -\frac{2\mu}{r^{1/2}}$

Multiplying both sides with $2r^2dr$ and Integrating, we get

$$2r^3 v\,dv + 3r^2v^2dr = -4\mu r^{3/2}dr$$

$$r^3v^2 = -\frac{8\mu}{5}r^{5/2} + B$$

Now $\quad r = c, \; v = 0$

$$\Rightarrow B = \frac{8\mu}{5}c^{5/2}$$

Rate of increase of mass within V

$$= \frac{d}{dt}\int_V \rho\, dv$$

By the Principle of continuity, we have
Accumulation = In − out + source − sink

$$\int_V \frac{\partial}{\partial t}(\rho\, dv) = -\int_V \nabla(\rho q)\, dv$$

$$\int_V \left[\frac{\partial \rho}{\partial t} + \nabla(\rho q)\right] dv = 0$$

This Relation is valid for all arbitrary volume V, Provided that it lies entirely in the fluid.

Thus the integrand must be identically zero every where in the fluid

$$\frac{\partial \rho}{\partial t} + \nabla(\rho q) = 0$$

at all Point in the region

$$\frac{\partial \rho}{\partial t} + q(\nabla \rho) + \rho \nabla q = 0$$

$$\left(\frac{\partial}{\partial t} + q\nabla\right)\rho + \rho\nabla q = 0$$

this is Eqn. $\boxed{\frac{D\rho}{Dt} + \rho \nabla q = 0}$ of continuity

which is valid at any Point of a fluid free form source and sinks.

This is also known as the Eqn of Kinematical relation. This is relative between velocity and density field.

$$\frac{D}{Dt} \equiv 0$$

For a steady flow of fluid, the Pattern of flow does not very regard

$$\frac{A(x^2-y^2)}{(x^2+y^2)^2} \cdot \frac{2Ax(3y^2-x^2)}{(x^2+y^2)^3} - \frac{2Axy}{(x^2+y^2)^2} \cdot 2Ay\frac{(3x^2-y^2)}{(x^2+y^2)^3}$$

$$= -\frac{1}{\rho}\frac{\partial P}{\partial x}$$

$$\frac{1}{\rho}\frac{\partial P}{\partial x} = \frac{2A^2x}{(x^2+y^2)^3}$$

$$\frac{1}{\rho}\frac{\partial P}{\partial y} = \frac{2A^2y}{(x^2+y^2)^3}$$

$$dP = \frac{\partial P}{\partial x}dx + \frac{\partial P}{\partial y}dy$$

So we can write

$$dP = \frac{2A^2\rho x}{(x^2+y^2)^3}dx + \frac{2A^2\rho y}{(x^2+y^2)^3}dy$$

$$= 2A^2\rho \frac{x\,dx + y\,dy}{(x^2+y^2)^3}$$

$$x^2+y^2 = t^2$$

$$2x\,dx + 2y\,dy = 2t\,dt$$

$$\Rightarrow x\,dx + y\,dy = t\,dt$$

$$dP = 2A^2\rho \frac{t\,dt}{t^5}$$

$$P = \frac{1}{2}\frac{A^2\rho}{(x^2+y^2)^2} + B$$

Constant of integration vanishes as x and y intially when $P = 0$ Ans

— x —

Each term represent a rate for the differen-tial element of volume.

consider a fluid element of infinitesimal volume δV and density ρ which is situated at a point r at any time t

mass of the fluid $= \rho \delta V$

Material derivative of the mass vanishes

$$\frac{D}{Dt}(\rho V) = 0$$

this is Eq^n of continuity in the simplest form

consider a closed surface S in a fluid medium containing a volume V fixed in space.

Let n is the unit outward drawn normal at a surface element δs. Let q is the fluid velocity at the surface element δs.

Then the normal component of the velocity q measured out ward will be

Rate of mass flow across $\delta s = \rho (n q) \delta s$

Total rate of mass flow $= \int_S \rho (n \cdot q) ds$

Total rate of mass flow in to $V = -\int_S \rho (n q) ds$

Using Gauss theorem, we have

$$= -\int_V \nabla (\rho q) dV$$

Total mass with in $V = \rho dV$